Modeling the Daytime Sky

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Abstract

The study of the appearance of the daytime sky and light scattering in the atmosphere generally requires complex numerical methods to model accurately. However, in cases where the atmosphere is optically thin, simple models can produce computationally inexpensive yet useful results. We present here a single-scatter model using a method of direct integration and present results which demonstrate the broadest features of the daytime sky.

Model and Method

Starting with an incident beam of light projected from the sun, a path is travelled from the top of the atmosphere to the ground. Attenuated light is scattered at every point along that path towards an observer as shown in **Figure 1.** From each scattering the point the light is attenuated along the path to the observer. This light will be affected by the angle between its initial path and final path by a function of that scattering angle. The integrated intensity is given by the expression:

$$I_g = I_o f(\psi) e^{-\frac{\tau}{\mu}} \frac{\mu_o}{\mu_o - \mu}\left(1 - e^{-\tau\left(\frac{1}{\mu}-\frac{1}{\mu_o}\right)}\right)$$

Where the phase function is given for scattering by a gas (Rayleigh scattering) here.

$$f(\psi) = \frac{3}{16\pi}(1 + cos^2\psi)$$

In the first equation the physical height of the atmosphere and the scattering characteristics of the atmosphere are combined into an *optical depth* τ. The angle between zenith and the observation direction is contained in the expression $\mu = cos(\theta)$.

The above expressions apply directly to a scalar model of light scattering which neglects the polarized nature of electromagnetic waves. But by replacing the scalar intensity with the Stokes vector which contains information about the state of a wave we may write

$$\bar{I} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}$$

And treat Rayleigh scattering by adding a matrix operation to scattering which operates on the Stokes vector using the Raleigh phase matrix,

$$M = \begin{pmatrix} 1 & -\left(\frac{1-cos^2(\psi)}{1+cos^2(\psi)}\right) & 0 & 0 \\ -\left(\frac{1-cos^2(\psi)}{1+cos^2(\psi)}\right) & 1 & 0 & 0 \\ 0 & 0 & \frac{2cos(\psi)}{1+cos^2(\psi)} & 0 \\ 0 & 0 & 0 & \frac{2cos(\psi)}{1+cos^2(\psi)} \end{pmatrix}$$

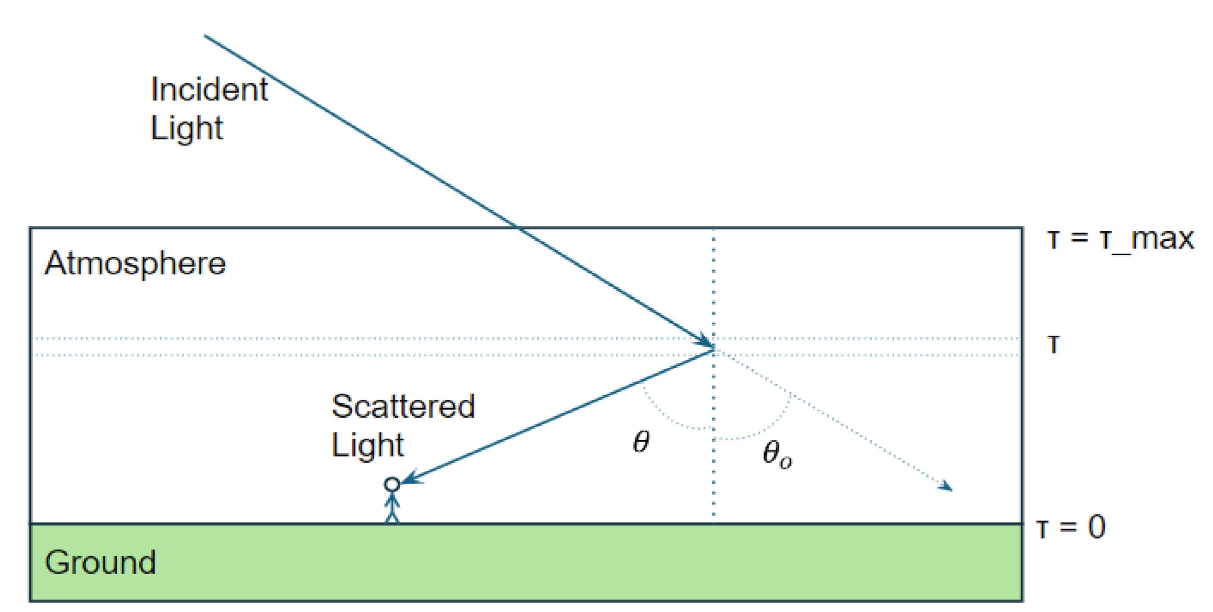

Figure 1. Model of the light interaction with the atmosphere

Qualitative Results

The distinct characteristics of a cloudless daytime sky can be observed in **Figure 2**. This figure is a fish-eye lens view of the entire sky where the center is straight up, and the edge of the figure is the horizon. The brightness of the sky is linearly scaled so show the broad features of limb brightening near the horizon and the relative darkness of the deep sky away from the sun as expected by Rayleigh scattering from the atmosphere. The solar position in the sky is shown by an inlaid yellow representation. These results were produced by calculating the expressions of the previous section in a Python script.

When observing the polarized sky in **Figure 3** as well as considering the position of the solar zenith angle it is easy to observe that the degree of polarization is its greatest about 90 degrees from the sun. This polarization is another observed result of Rayleigh scattering by atmospheric gases.
The value of a direct integration model is in its simplicity and speed of computation, combined with the fact that it captures so many important features of the observed light field. It is easy to construct images to confirm the qualitative agreement of the model to observation.

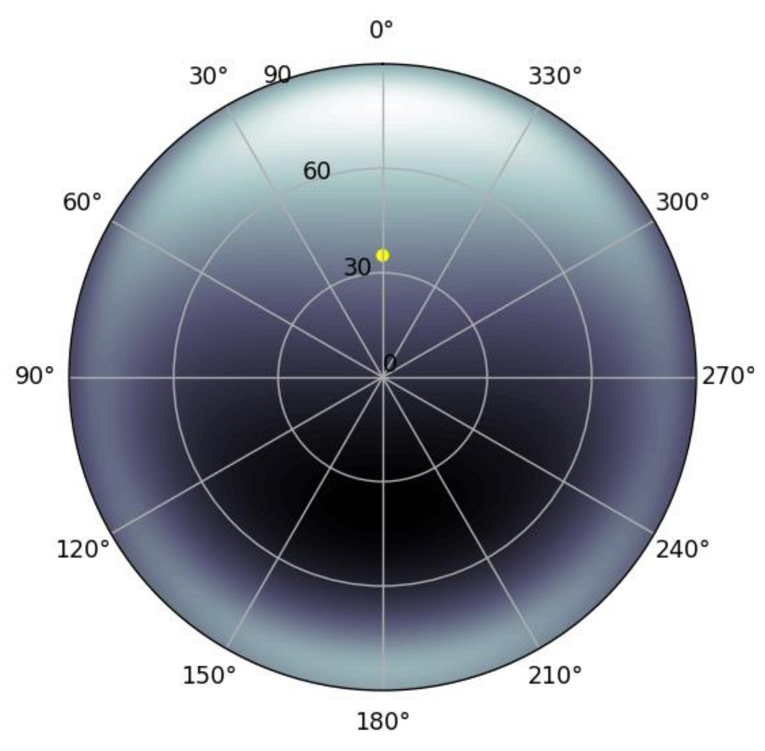

Figure 2. Intensity of Light.

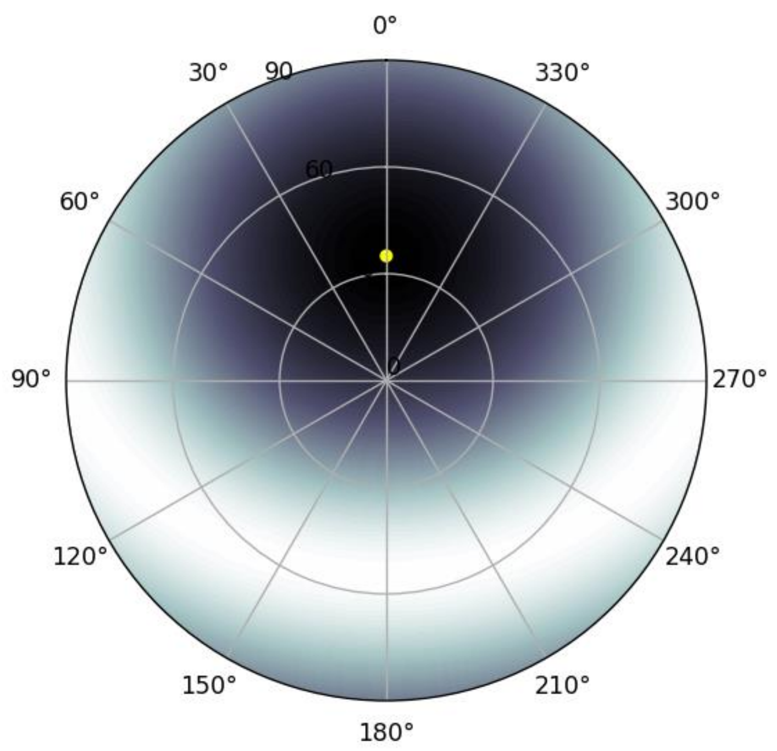

Figure 3. Degree of Polarization.

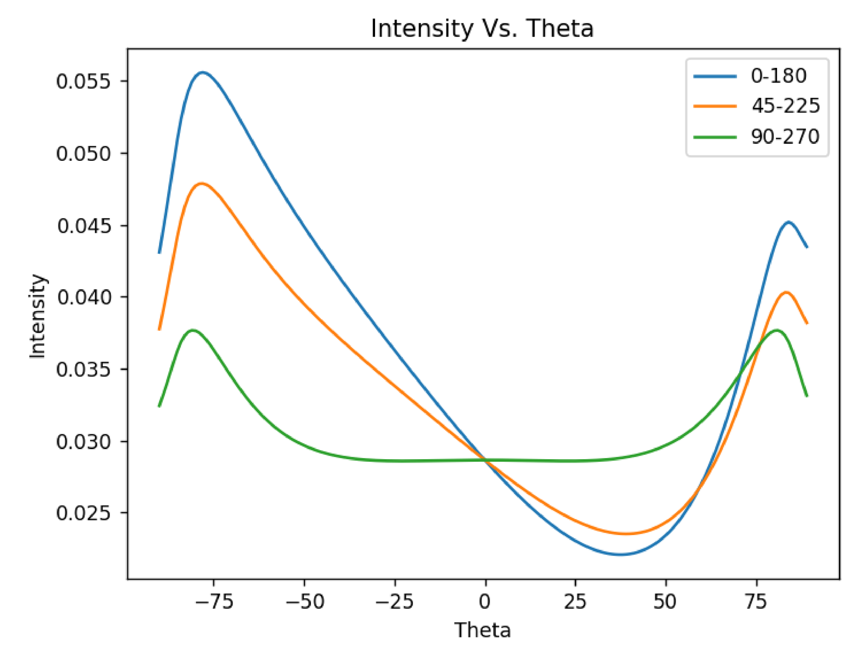

Figure 4. Graph of Intensity Vs. Theta

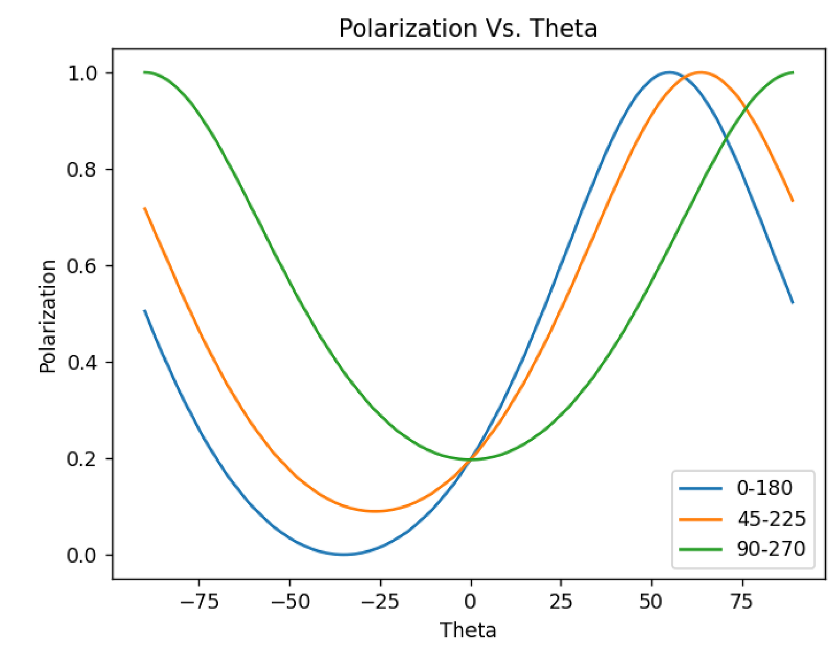

Figure 5. Graph of Polarization Vs. Theta

Numerical Results

There is a difference in what we can calculate between a scalar and a vector model with our direct integration method. Using a scalar model, we can produce the intensity of light in the atmosphere which the human eye may observe, but only with a vector model are we able to calculate the polarization observed with instrumentation or simple polarizers. The value the intensity calculated is shown in **Figure 4**, and this result is the same when using a scalar or vector model. **Figure 5** shows the values of the degree of polarization observed when using a vector model. The relation of the degree of polarization to the Stokes vector element is given by

$$P = \frac{\sqrt{Q^2 + U^2}}{I}$$

The curves in Figures 4 and 5 are taken along azimuth lines over a range of observer zenith angles from horizon to horizon.

In **Figure 4** one sees the effect of limb brightening on the intensity as one approaches the horizon at all azimuth angles. In **Figure 5** one can observe the high degree of polarization (100% in the case of single scattering) for scattering angles of 90 degrees from the incident solar beam. One also sees that in the deep dark region away from the sun that the polarization is quite great while near to the sun the polarization will be very low. The direct beam of the sunlight has zero polarization since there is no preferred orientation for light from the sun until it interacts with a surface or the atmosphere.

Conclusions

We have shown the results of a single-scatter model and demonstrated that it produces useful results when simulating the intensity and polarization of the light field in a daytime sky. Qualitatively this model reproduces the major observed features in intensity and polarization for an observer on the ground under a cloudless sky.

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