Compressed Sensing in Radar Applications Using Convex Optimization and Random Sampling

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Abstract

This poster aims to provide an overview of compressive sensing and its developments in radar applications. Conventional radar imaging techniques require acquiring many measurements to reconstruct the scene [1]. The advantages of compressed sensing, or compressed sampling, are low energy consumption, high-speed measurements, and revolutionary data acquisition [2][3]. There are many techniques for solving the compressed sensing problem. Chaotic frequency signals present advantages by having a wider bandwidth, contriving noise, and easy generation using a Bernoulli map [4], and they can be used in compressive radar. Using MATLAB, a radar scene was simulated, and compressive sensing techniques were implemented. The disciplined convex programming algorithm CVX was used to reconstruct the radar scene [5] from the simulated radar measurements. Convex programming provides the most accurate results when reconstructing data at the cost of high computational complexity. In addition to CVX, other greedy algorithms can be used to solve a radar signal's compressed sensing problem that requires further understanding [6].

Introduction

Compressive sensing is a signal processing technique that efficiently measures and reconstructs data by finding solutions to an underdetermined system of linear equations. The issue with an underdetermined system is that there are infinitely many solutions. One of the conventional methods is to use the Shannon Nyquist sampling rate. The theorem states, "measurements must be taken at twice the rate of the maximum frequency," known as the Nyquist rate [1]. However, the frequency of the signals may not be known or chaotically modulated. Furthermore, taking measurements at a high rate of speed creates a higher cost in data and power consumption. Other optimization techniques are explored to solve the linear system

$$\boldsymbol{y} = \boldsymbol{\Theta s} \quad (1)$$

$\boldsymbol{y}$ is the received signal, and $\boldsymbol{\Theta}$ is the measurement matrix shown in Figure 1 and Figure 2. Solving for the sparse vector $\boldsymbol{s}$ will give a unique reconstruction of $\boldsymbol{y}$. To satisfy the optimization problem, $\boldsymbol{s}$ must have the sparsest solution. A sparse solution has very few non-zero elements compared to its dimension. It is possible to collect dramatically fewer measurements that are randomly sampled and then solve for the non-zero elements in $\boldsymbol{s}$ [2]. Figure 3 shows the reduced measurement that appears more granular than Figure 1. Figure 4 demonstrates that random columns of the matrix can be removed and will not affect the reconstructed scene.

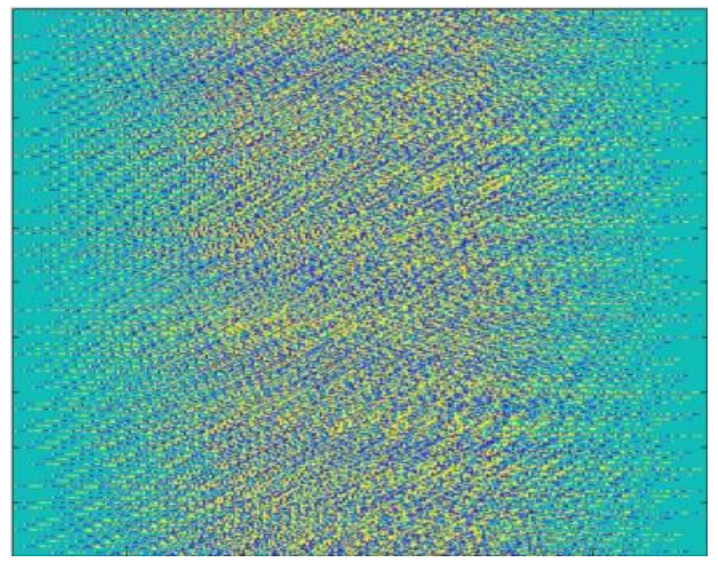


Figure 1. Measurement Matrix.

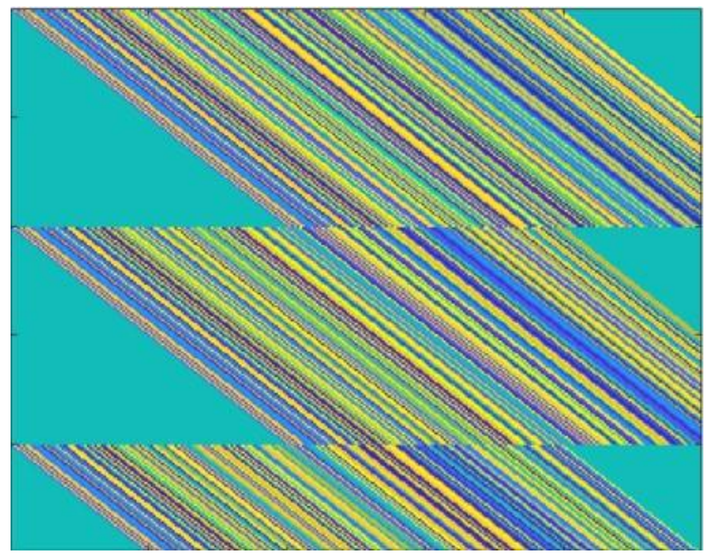


Figure 2. Zoomed in Matrix.

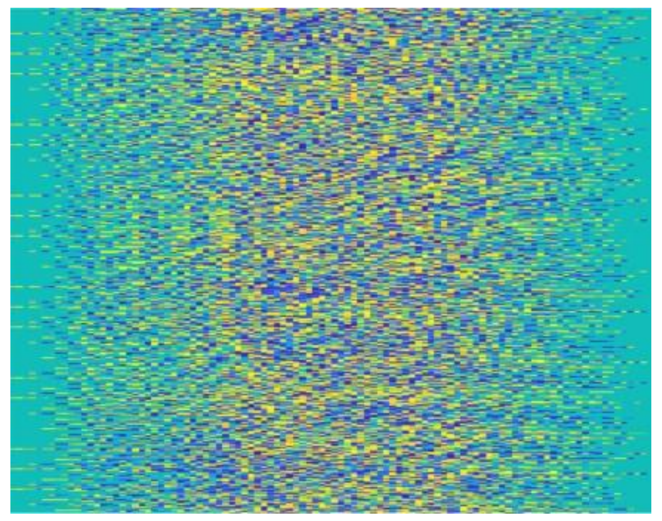


Figure 3. Reduced Measurement.

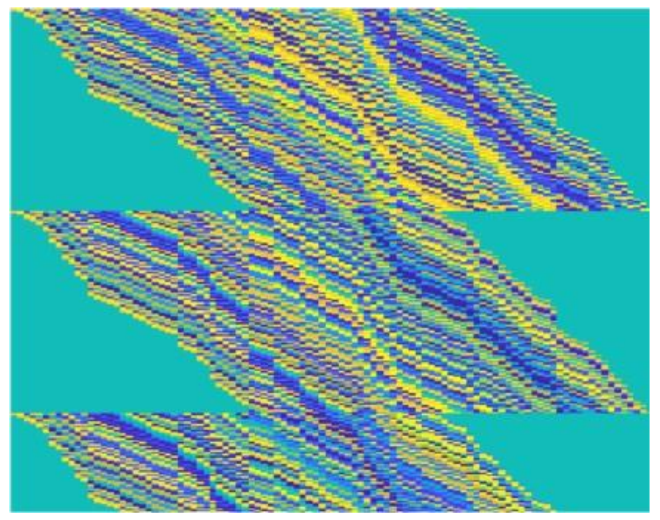


Figure 4. Zoomed in Matrix.

Discussion

Radar scenes are generally sparse and do not require any transformations in their domain before compression. Figure 5 shows the reconstructed radar scene that was measured. The scene is considerably sparse in its dimension when comparing the white signals to the black background. The white values can be assigned as non-zero entries in a sparse vector. Additionally, The measurements $\boldsymbol{\Theta}$ from (1) must also be incoherent with respect to the sparsity basis [3]. If these conditions are met, then the scene is easily compressible. Chaotic signals provide advantages when measuring a radar scene. Chaotic signals are easily generated through a random unique process, have a wide band frequency range, and behave like pseudo noise [4]. These prosperities make it more difficult for the targets to decode the signal being emitted from the radar. Figure 6 demonstrates the emitted signal reflected by the targets residing in the scene.

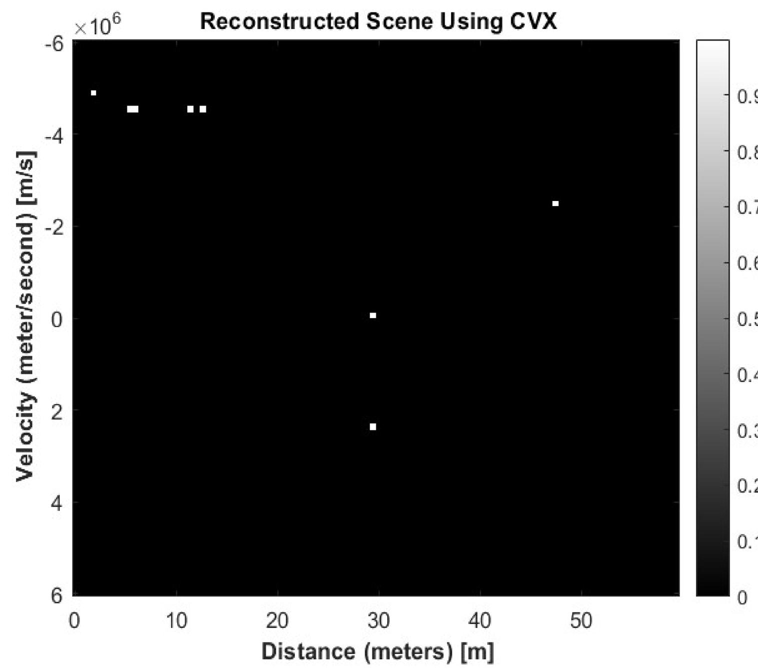


Figure 5.

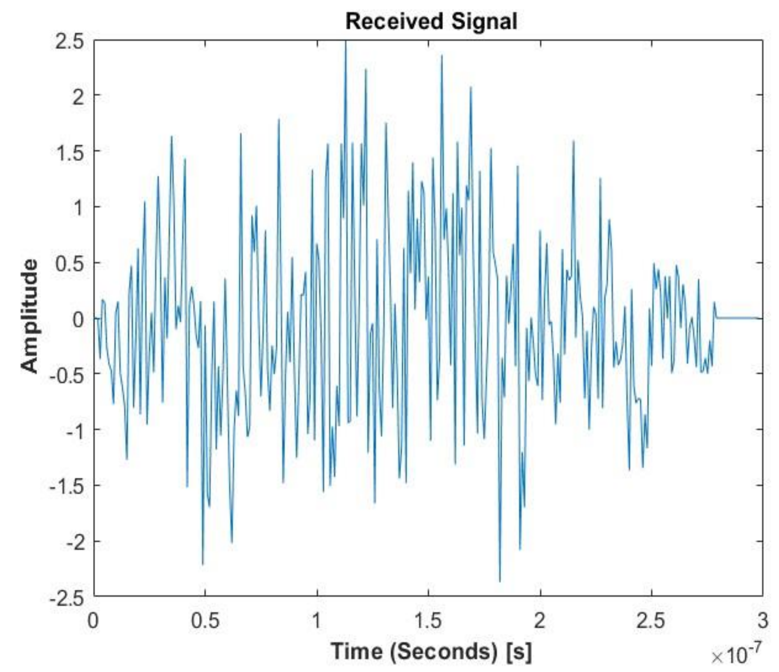


Figure 6.

Methods

Chaotic-based frequency modulated signals (CBFM) can be generated from several different chaotic maps. The Bernoulli map was chosen because it remained chaotic throughout its generation [4]. The generation of the map consists of choosing a random number between zero and one and subtracting 0.5. This initial value will build an iterative sum where each iterative component is stored in a vector x_k to build the signal $s(n)$.

$$s(n) = Ae^{(j2\pi K \sum_{k=0}^{N} x_k)} \quad (2)$$

Where A is amplitude, j is the imaginary number, and K is the modulation index. The sum will iterate until the length of the square measurement matrix. The generated signal and reduced measurement matrix allows the use of convex optimization can be used to solve the underdetermined system of linear equations [5]. Using (1) can be turned into a convex l_1 norm minimization problem [2].

$$\hat{\boldsymbol{s}} = \min_{s} \|\boldsymbol{s}\|_1 \text{ subject to } \boldsymbol{y} = \boldsymbol{\Theta s} \quad (3)$$

The sparse vector $\hat{\boldsymbol{s}}$ has the fewest nonzero entries and efficiently solves the compressed sensing problem. The signal can now be reconstructed with minimal accuracy loss given by the new equation.

$$\boldsymbol{y} = \boldsymbol{\Theta}\hat{\boldsymbol{s}} \quad (4)$$

Results

The original radar scene is identical to the reconstructed scene using CVX shown in Figure 5. The original scene was not included to save space for this presentation. Before using CVX, the scene was placed into a vector space like in Figure 7. The reconstructed vector is also identical to its original measurement. However, the CVX program is not perfect and will rebuild the scene within 95% accuracy. The match filter results can be used to reconstruct the scene. The filter compares the received signal on the convolution axis for each iteration and outputs correlation. The peaks in Figure 9 represent a high signal-to-noise ratio, indicating that a target is present in the measured setting. A negative correlation is not a detection of a target and represents an inverse correlation between signals.

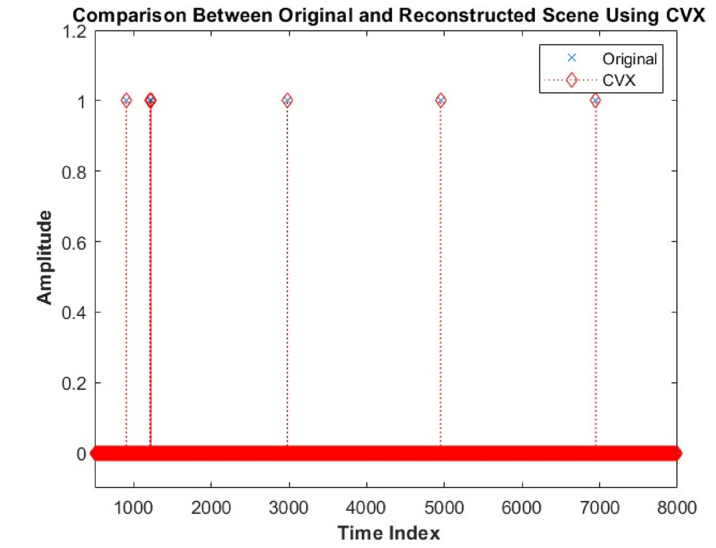


Figure 7.

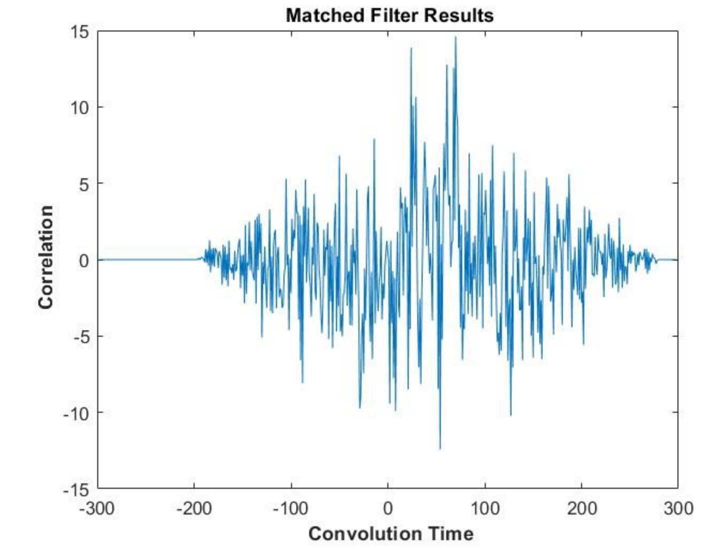
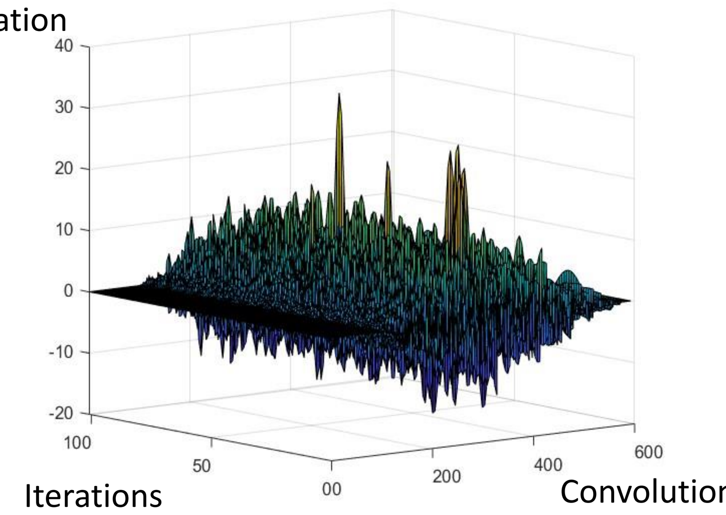


Figure 8.

Figure 9.
Represents a 3D surface of the matched Filter. Where each iteration can be represented as cross section of Figure 8



Conclusions

CVX solves the compressed sensing problem efficiently but at the cost of high computation from the computer processing the signal [6]. It is believed that other optimization algorithms exist that can solve these radar problems with the same amount of accuracy while also processing the signal at a faster rate. Chaotic signals are easy to generate and provide discreet detection of moving targets. Further research is required to understand other algorithms and compare them to CVX in simulation testing.

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References

1. E. J. Candès, M. B. Wakin, "An Introduction to Compressive Sampling," IEEE Signal Processing Magazine, March 2008
2. S. L. Brunton, J. N. Kutz, "Sparsity and Compressed Sensing," in Data Driven Science and Engineering 2017, Ch. 3, pp 96 – 125
3. M. Wisconsin, J. Romberg, M. Wakin, "Compressed Sensing: A Tutorial", IEEE Statistical Signal Processing Workshop, August 2007
4. A. Ashtari, G. Thomas, H. Garces, B. C. Flores, "Radar Signal Design Using Chaotic Signals" IEEE Xplore, Waveform Diversity and Design, July 2007
5. I. Orovic, V. Papic, C. Ioana, X. Li, S. Stankovic, "Compressive Sensing in Signal Processing: Algorithms and Transform Domain Formulations," IEEE Xplore, August 2016
6. Michael Grant and Stephen Boyd. CVX: MATLAB software for disciplined convex programming, version 2.0 beta. http://cvxr.com/cvx, September 2013.